Problem Solving Strategies Workshop

Problem solving

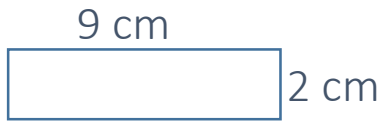
....is knowing what to do when you don’t know what to do

Why do you need to be good at problem solving?

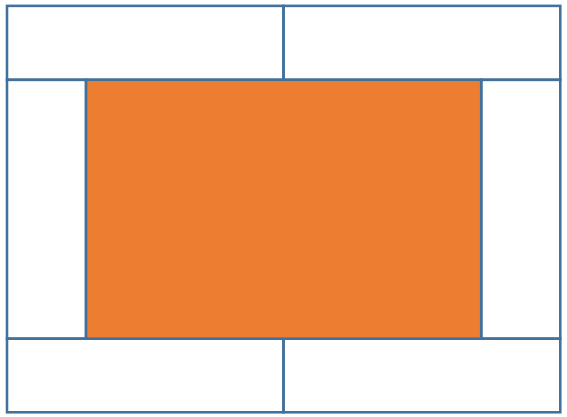
- It is what mathematics is all about.
- Encourages and develops important life skills, such as creative thinking and risk-taking.
- Research shows that students who are confident in problem solving often do better in class work and exams, as they are not fazed by questions they don't immediately know how to answer.
- **50% of the new Foundation GCSE** exams and **60% of the new Higher GCSE** exams is problem solving.
- So many careers involve problem solving. Can you think of some examples?

Try this:

Here is a 9 cm by 2 cm rectangle.



Six of these rectangles are used to make this pattern.



Work out the area that is shaded.

How many ways can you do this?

How did you decide what to do?

Which is easiest? Why?

How could you make the problem harder?

Today's objectives:

- To become better problem solvers by applying Problem Solving Strategies to GCSE exam questions.

By the end of the session, you will have practised how to use the following strategies to solve a range of problems:

✓ Draw pictures and diagrams to represent the problem

✓ Use bar models

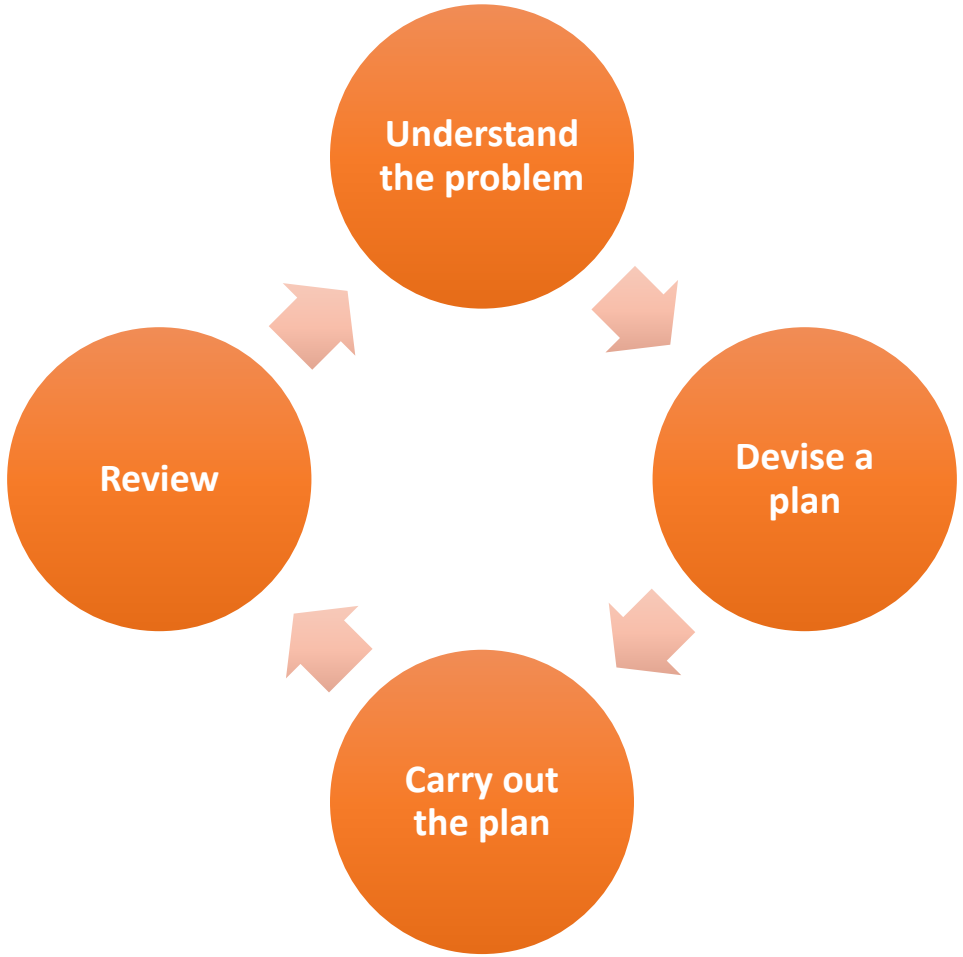
✓ Use a table to look for patterns

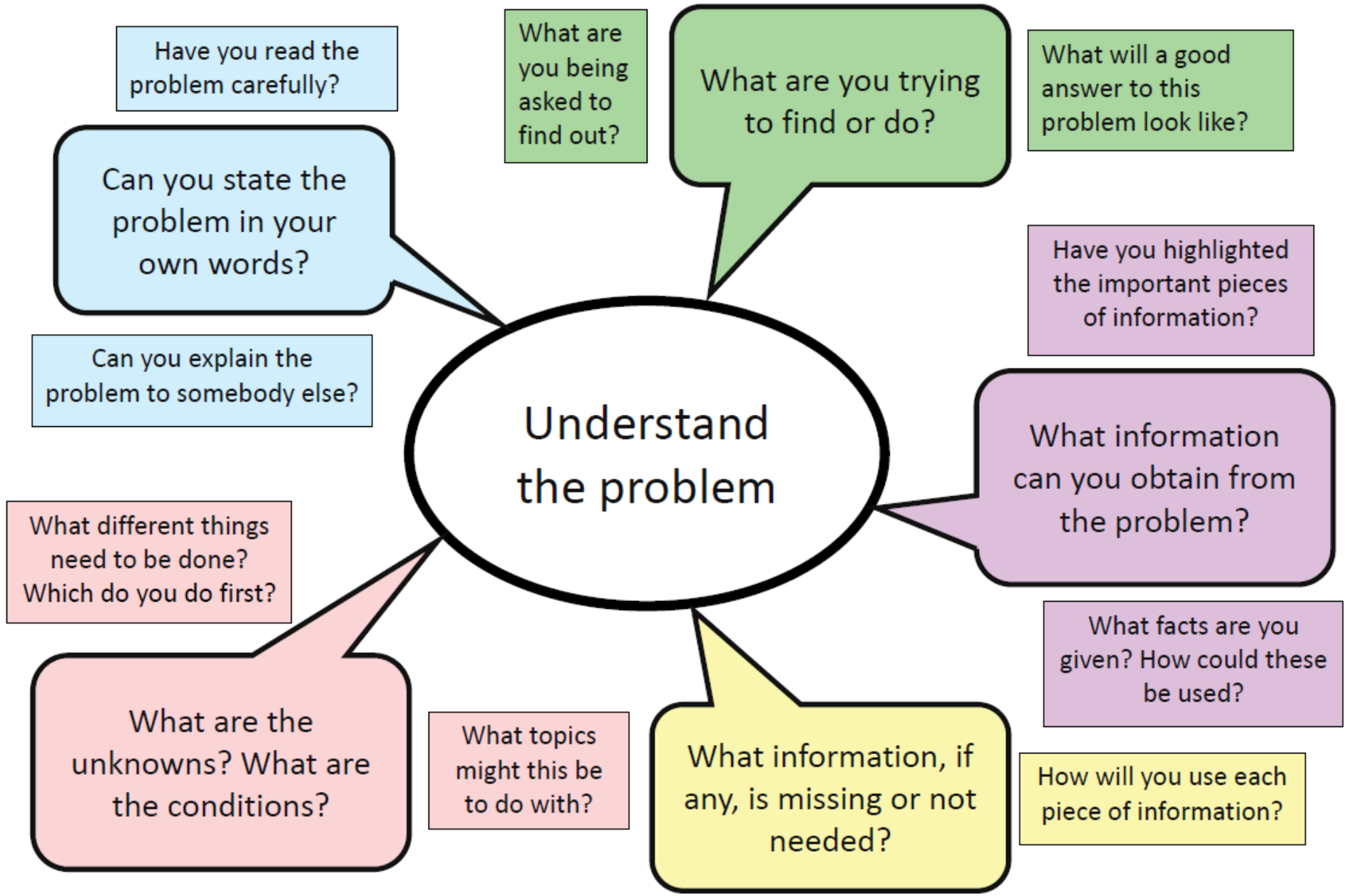
✓ Set up an equation (use x for the unknown)

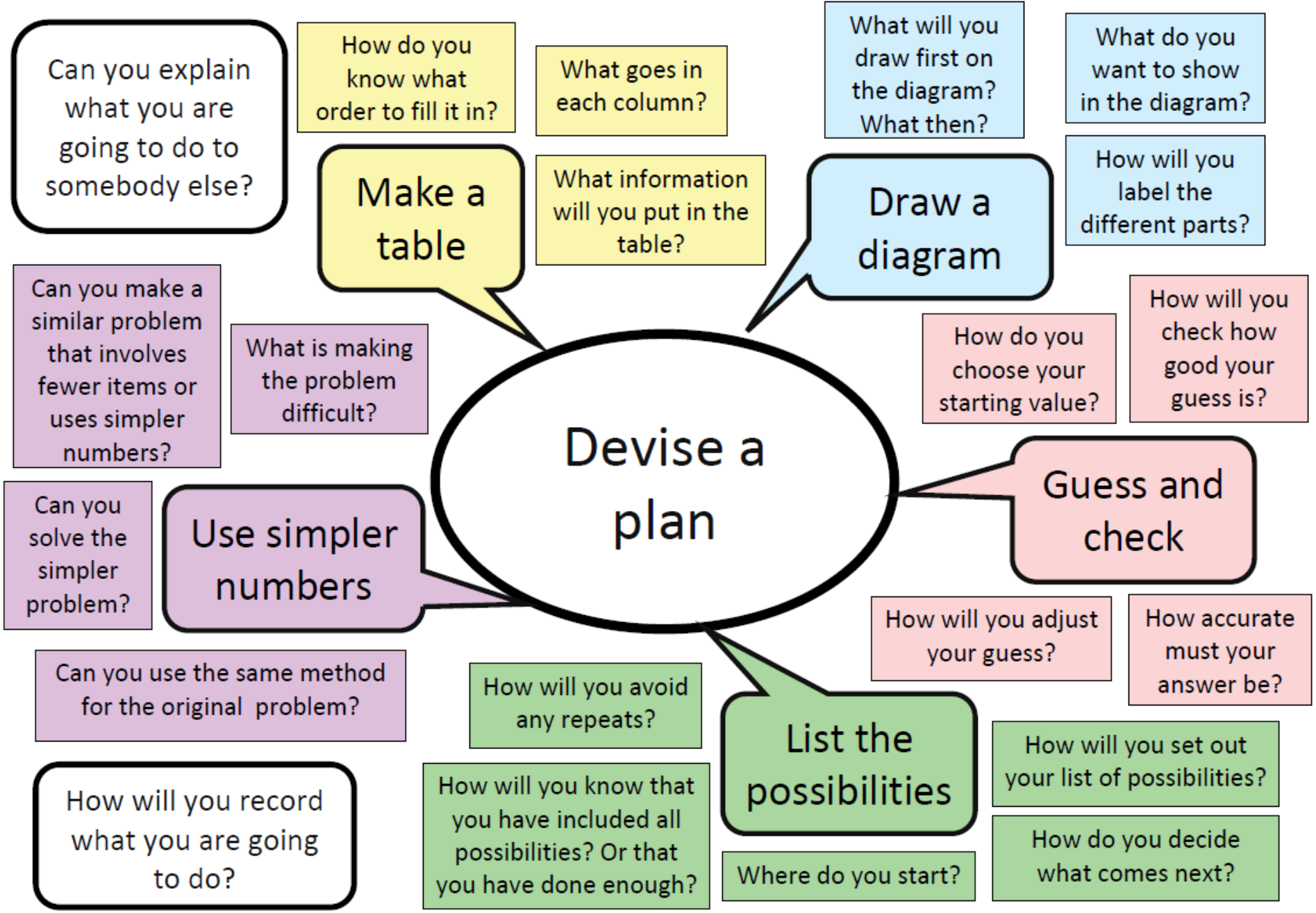
✓ Work systematically by drawing arrow diagrams

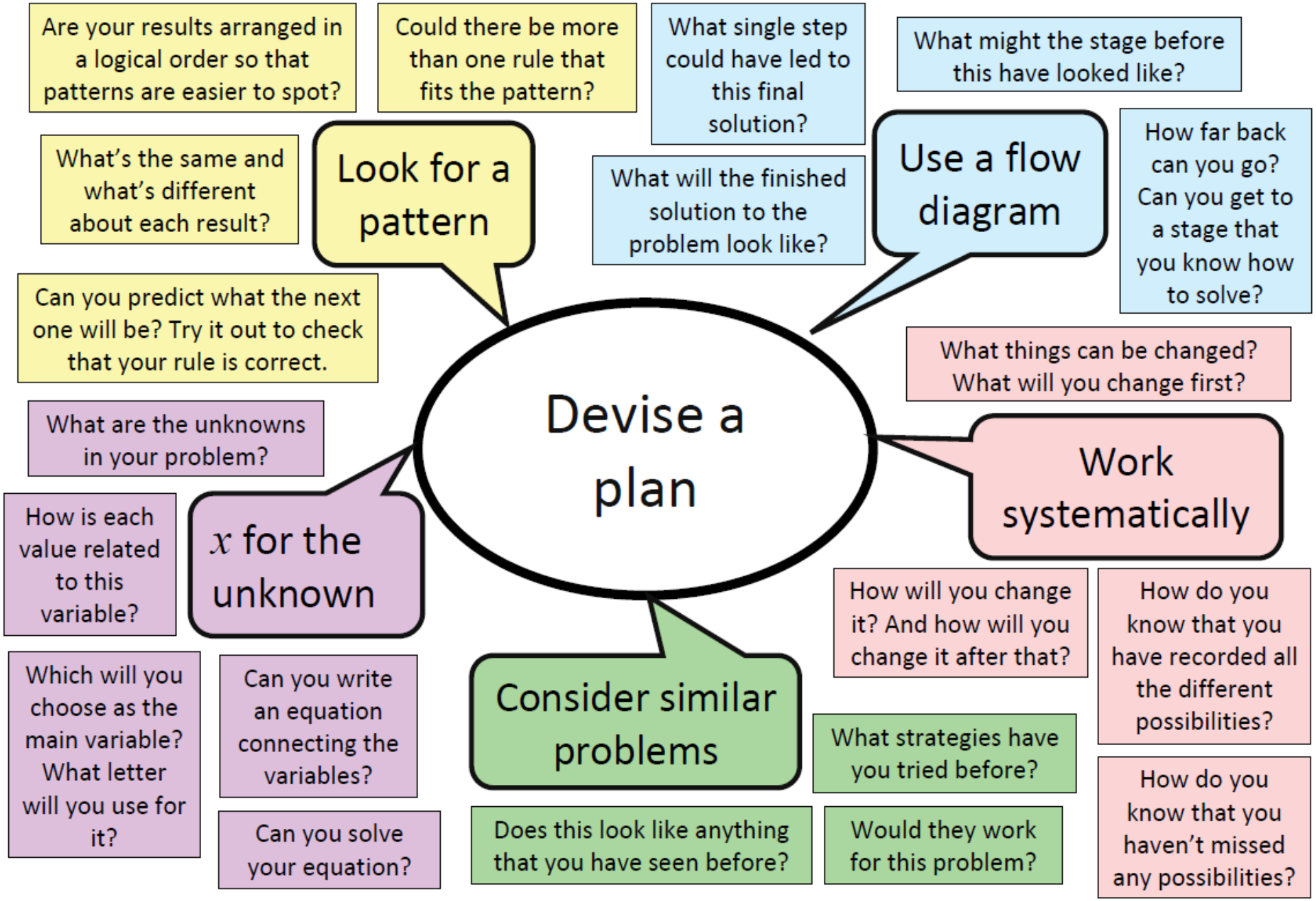
The problem-solving phases

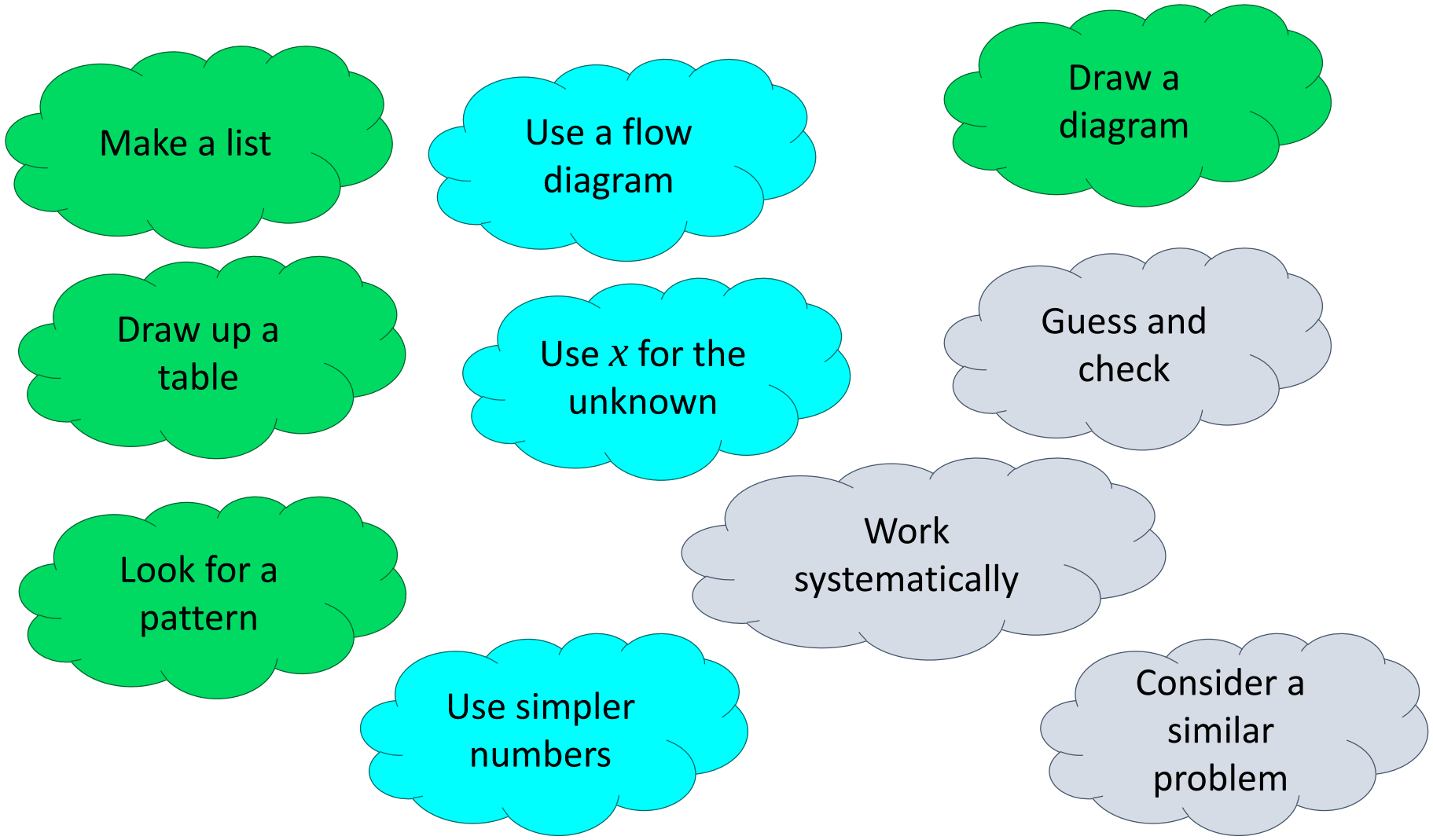
When you have to solve a problem, following these phases will help.

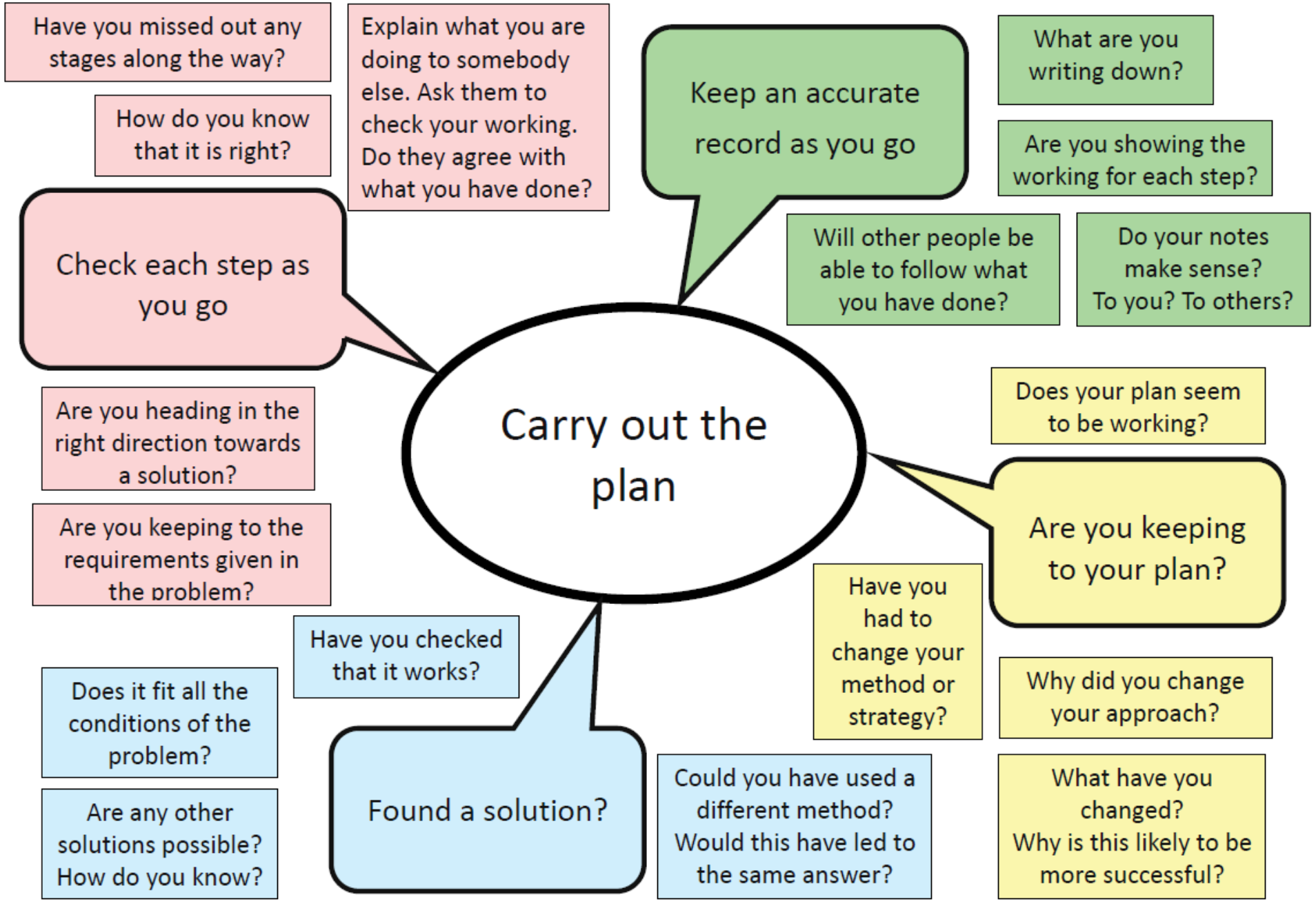


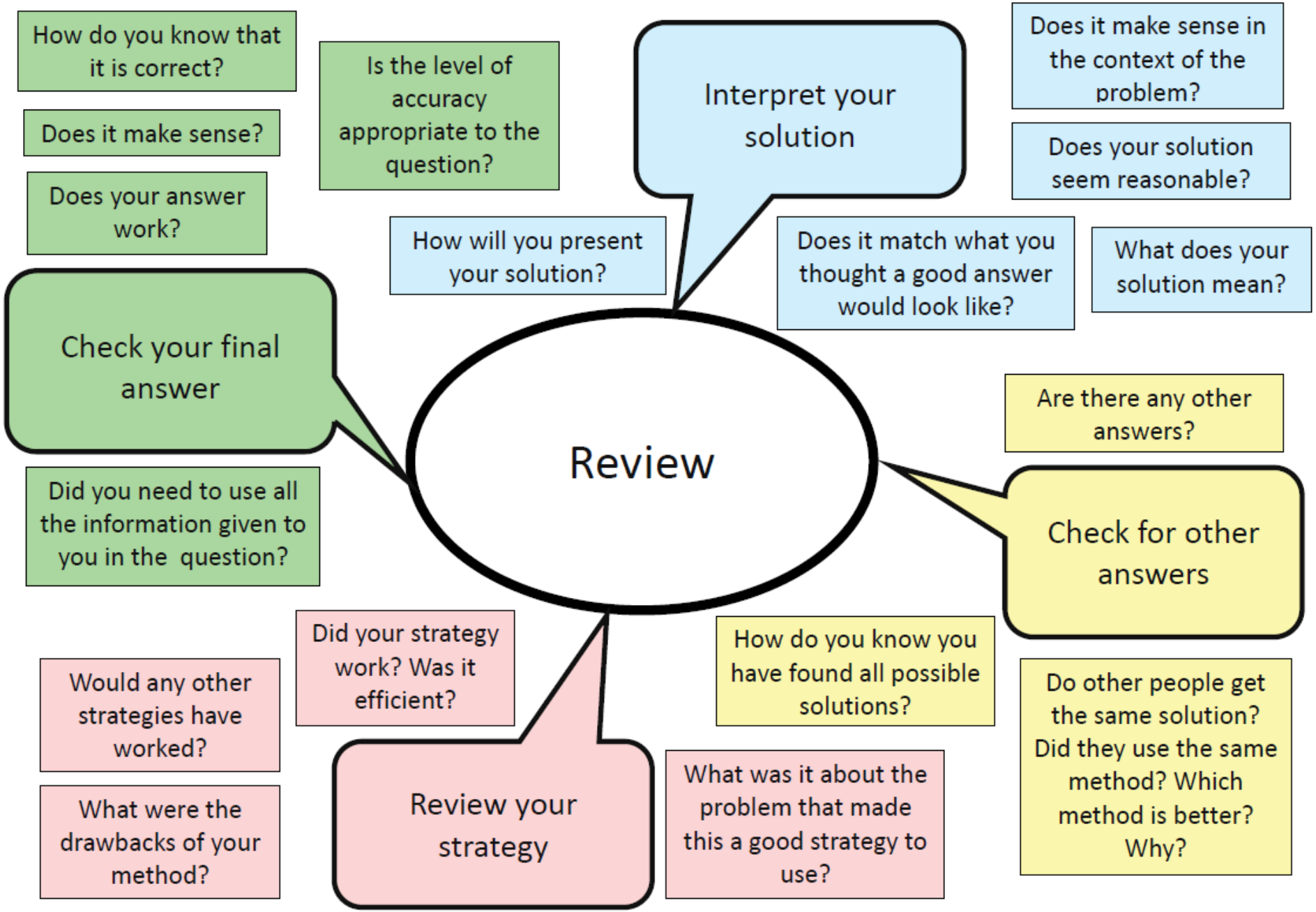

Understand the problem
Have you read the problem carefully?
Can you state the problem in your own words?
Can you explain the problem to somebody else?
What are you being asked to find out?
What are you trying to find or do?
What will a good answer to this problem look like?
Have you highlighted the important pieces of information?
What information can you obtain from the problem?
What facts are you given? How could these be used?
How will you use each piece of information?
What information, if any, is missing or not needed?
What topics might this be to do with?
What are the unknowns? What are the conditions?
What different things need to be done? Which do you do first?


Devise a plan
Make a table
How do you know what order to fill it in?
What goes in each column?
What information will you put in the table?
Draw a diagram
What will you draw first on the diagram? What then?
What do you want to show in the diagram?
How will you label the different parts?
Guess and check
How do you choose your starting value?
How will you check how good your guess is?
How will you adjust your guess?
How accurate must your answer be?
List the possibilities
How will you avoid any repeats?
How will you know that you have included all possibilities? Or that you have done enough?
Where do you start?
How will you set out your list of possibilities?
How do you decide what comes next?
Use simpler numbers
Can you make a similar problem that involves fewer items or uses simpler numbers?
What is making the problem difficult?
Can you solve the simpler problem?
Can you use the same method for the original problem?
Can you explain what you are going to do to somebody else?
How will you record what you are going to do?

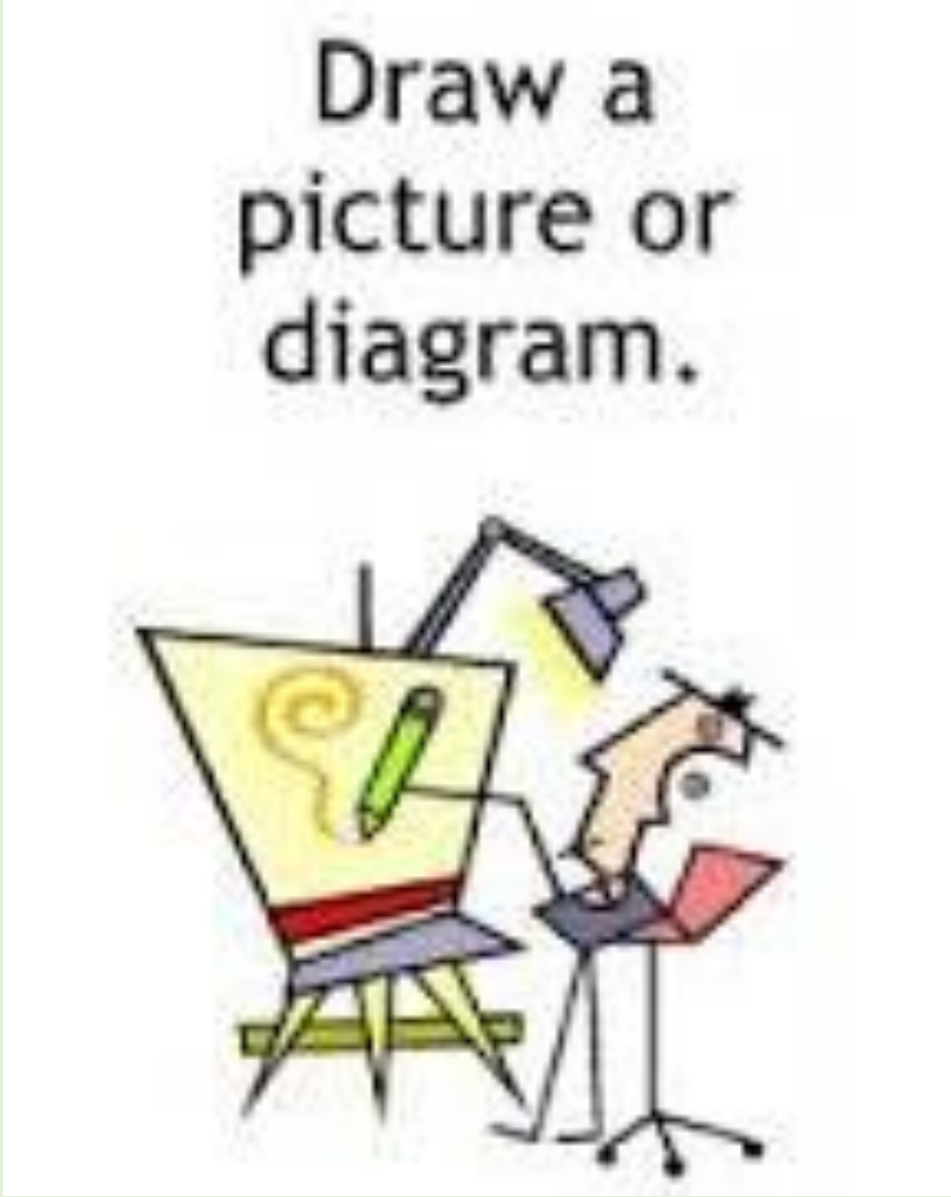

Are your results arranged in a logical order so that patterns are easier to spot?
Could there be more than one rule that fits the pattern?
What single step could have led to this final solution?
What might the stage before this have looked like?
What's the same and what's different about each result?
Look for a pattern
What will the finished solution to the problem look like?
Use a flow diagram
How far back can you go? Can you get to a stage that you know how to solve?
Can you predict what the next one will be? Try it out to check that your rule is correct.
What things can be changed? What will you change first?
Devise a plan
What are the unknowns in your problem?
How is each value related to this variable?
x for the unknown
Work systematically
How will you change it? And how will you change it after that?
How do you know that you have recorded all the different possibilities?
Which will you choose as the main variable? What letter will you use for it?
Can you write an equation connecting the variables?
Consider similar problems
What strategies have you tried before?
How do you know that you haven't missed any possibilities?
Can you solve your equation?
Does this look like anything that you have seen before?
Would they work for this problem?


Make a list
Use a flow diagram
Draw a diagram
Draw up a table
Use x for the unknown
Guess and check
Look for a pattern
Work systematically
Use simpler numbers
Consider a similar problem


Carry out the plan
Check each step as you go
Have you missed out any stages along the way?
How do you know that it is right?
Explain what you are doing to somebody else. Ask them to check your working. Do they agree with what you have done?
Keep an accurate record as you go
What are you writing down?
Are you showing the working for each step?
Will other people be able to follow what you have done?
Do your notes make sense? To you? To others?
Are you keeping to your plan?
Does your plan seem to be working?
Are you heading in the right direction towards a solution?
Are you keeping to the requirements given in the problem?
Have you had to change your method or strategy?
Why did you change your approach?
Found a solution?
Have you checked that it works?
Does it fit all the conditions of the problem?
Are any other solutions possible? How do you know?
Could you have used a different method? Would this have led to the same answer?
What have you changed? Why is this likely to be more successful?


Review
Interpret your solution
Does it make sense in the context of the problem?
Does your solution seem reasonable?
What does your solution mean?
Does it match what you thought a good answer would look like?
How will you present your solution?
Check your final answer
How do you know that it is correct?
Is the level of accuracy appropriate to the question?
Does it make sense?
Does your answer work?
Did you need to use all the information given to you in the question?
Check for other answers
Are there any other answers?
How do you know you have found all possible solutions?
Do other people get the same solution? Did they use the same method? Which method is better? Why?
Review your strategy
Did your strategy work? Was it efficient?
What was it about the problem that made this a good strategy to use?
Would any other strategies have worked?
What were the drawbacks of your method?

Strategy 1



Drawing Diagrams 1

Draw a diagram to help solve this problem.

A circus clown climbs a 16 m slippery pole.

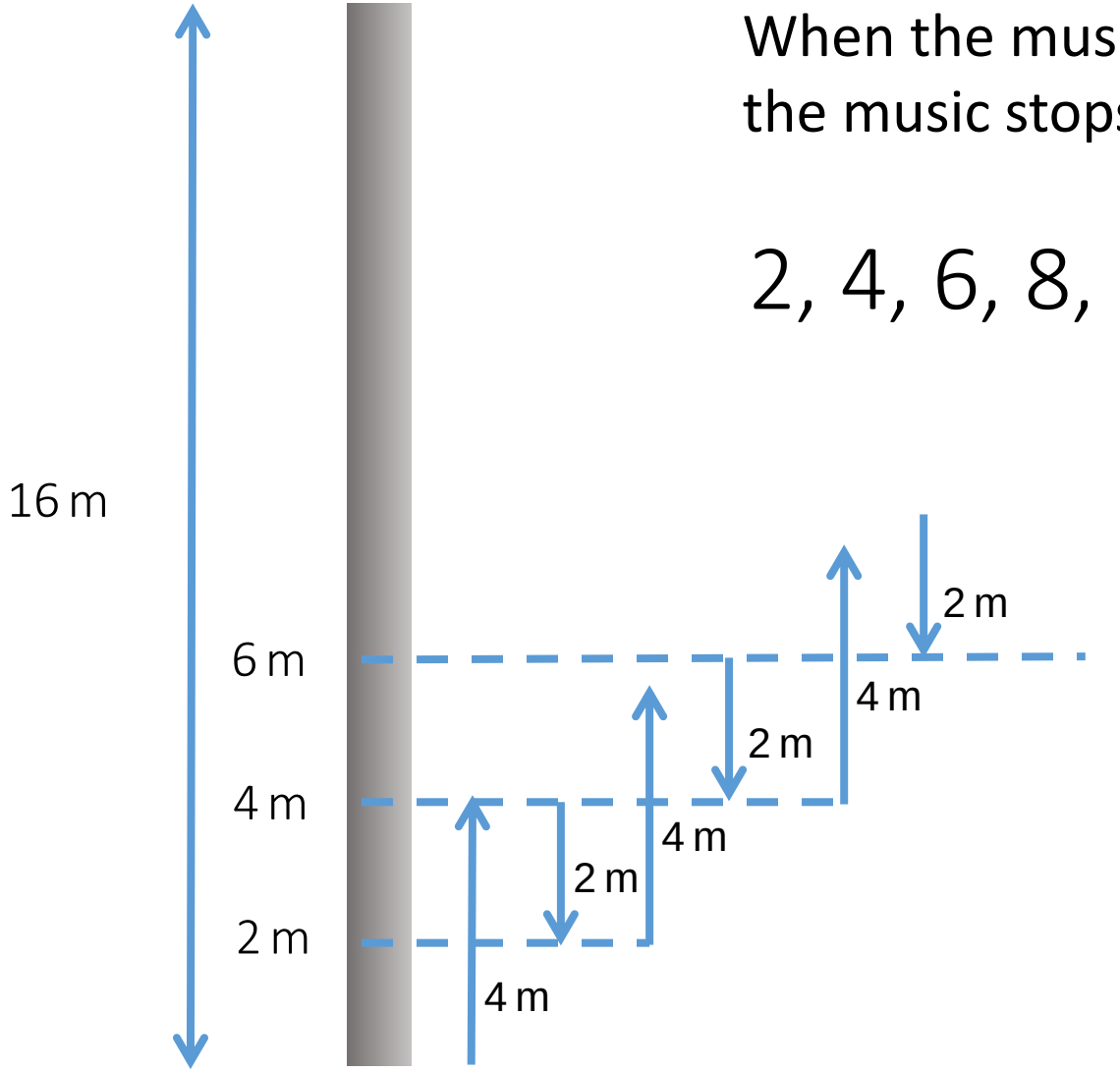
When the music plays, he climbs up 4 m.

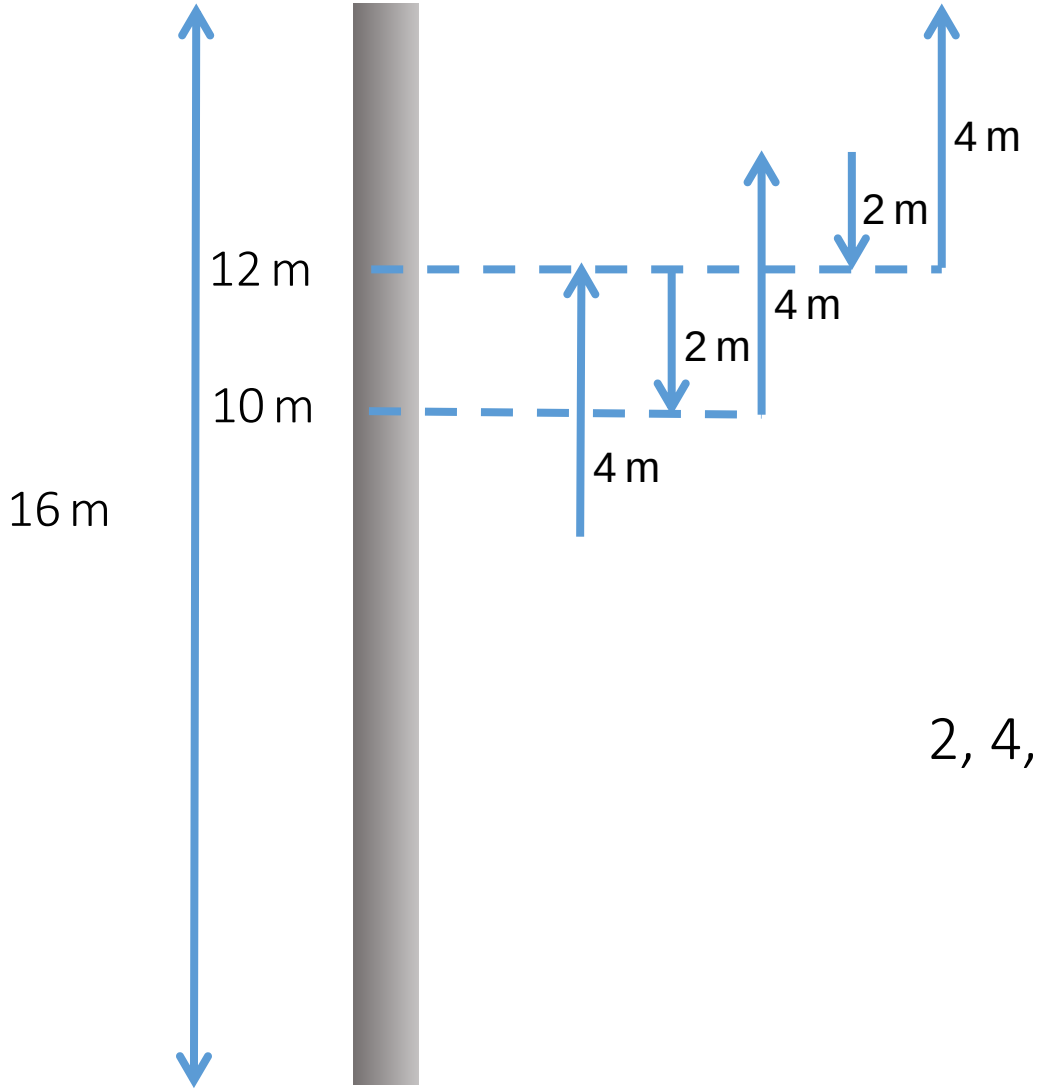
When the music stops, he slides down 2 m.

How many times must the music play for the clown to reach the top of the slippery pole?

When the music plays, he climbs up 4 m. When the music stops, he slides down 2 m.

2, 4, 6, 8, 10, 12, ...

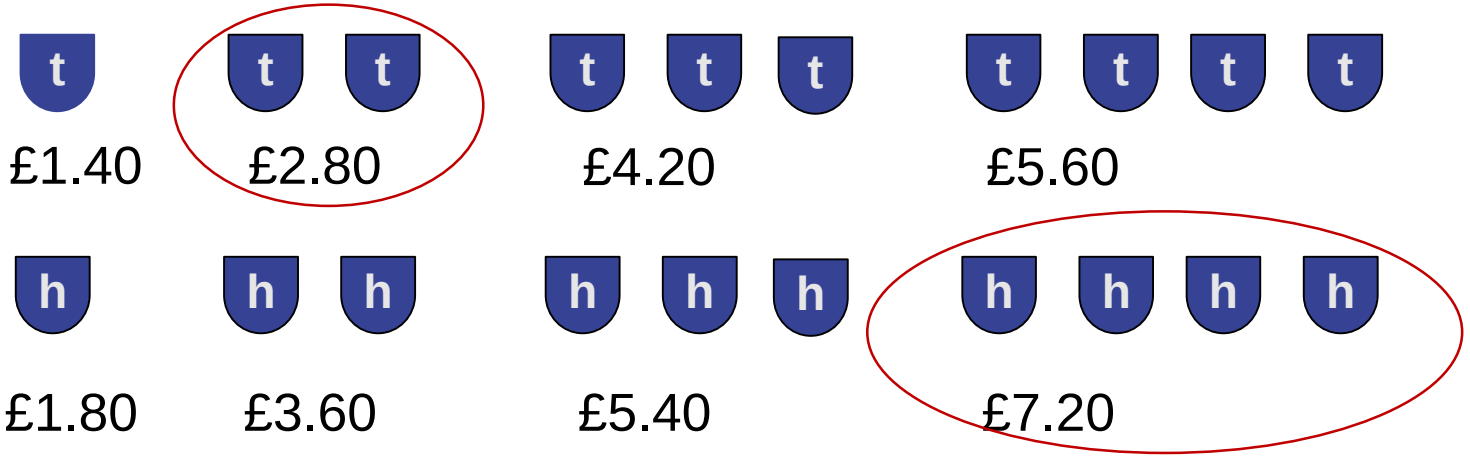




2, 4, 6, 8, 10, 12, …..

Drawing diagrams 2

In a café, tea costs £1.40 and hot chocolate costs £1.80.
A group of friends bought some drinks and paid £10.
How many of each kind of drink did they buy?



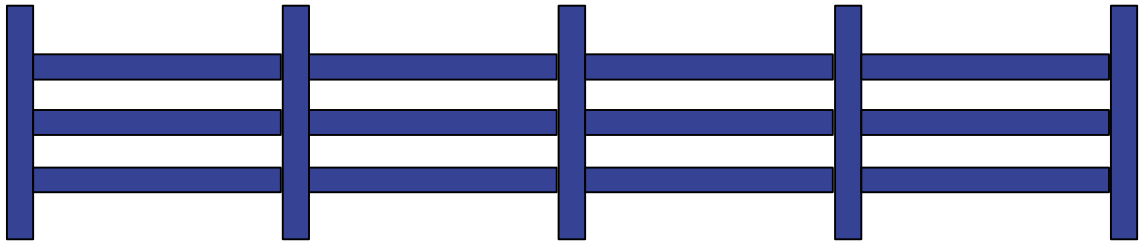
Look for a total number of teas and hot chocolates that together cost £10.

Solution:
2 teas and 4 hot chocolates

Other examples

A fence is made from 1.2 m lengths of wood. It has supporting posts joined by three horizontal beams to make a panel.

What length of wood is needed for a four panel fence?



Number of posts = 5

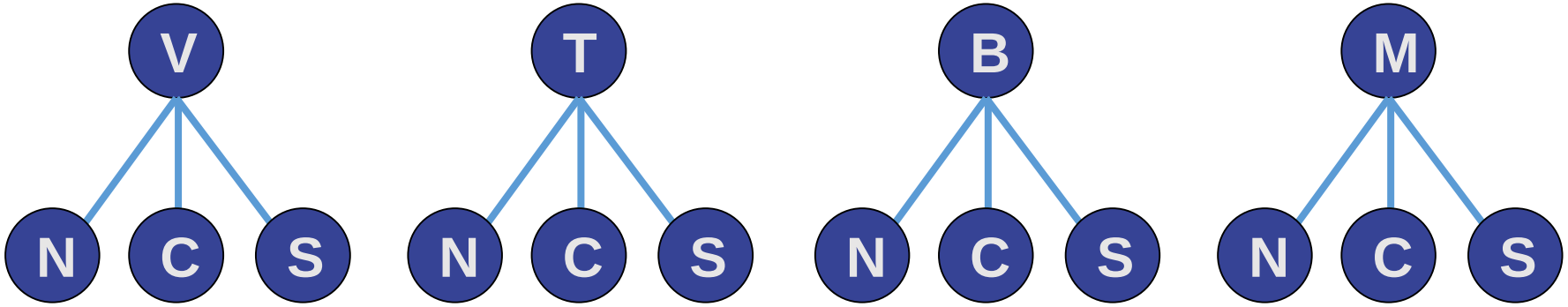
Number of beams = $4 \times 3 = 12$

Total = $5 + 12 = 17$

Total length = $17 \times 1.2 = 20.4$ m

More examples

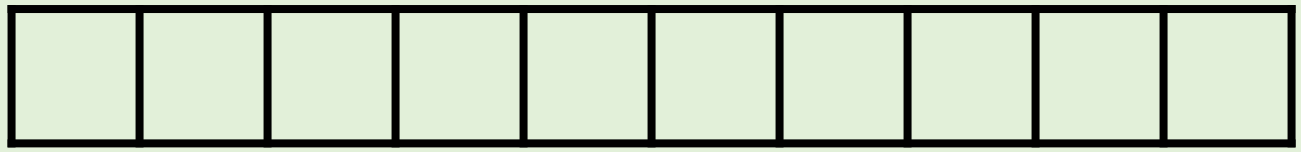
An ice cream seller offers vanilla, toffee, banana or mint ice cream.
He also offers a choice of nuts, chocolate chips or sprinkles for toppings.
How many different combinations of a single flavour ice cream with one topping can be ordered?



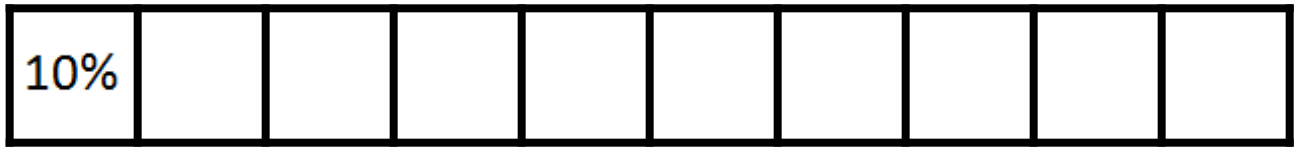
The number of combinations is $4 \times 3 = 12$

Strategy 2

Draw a bar model



Discuss:



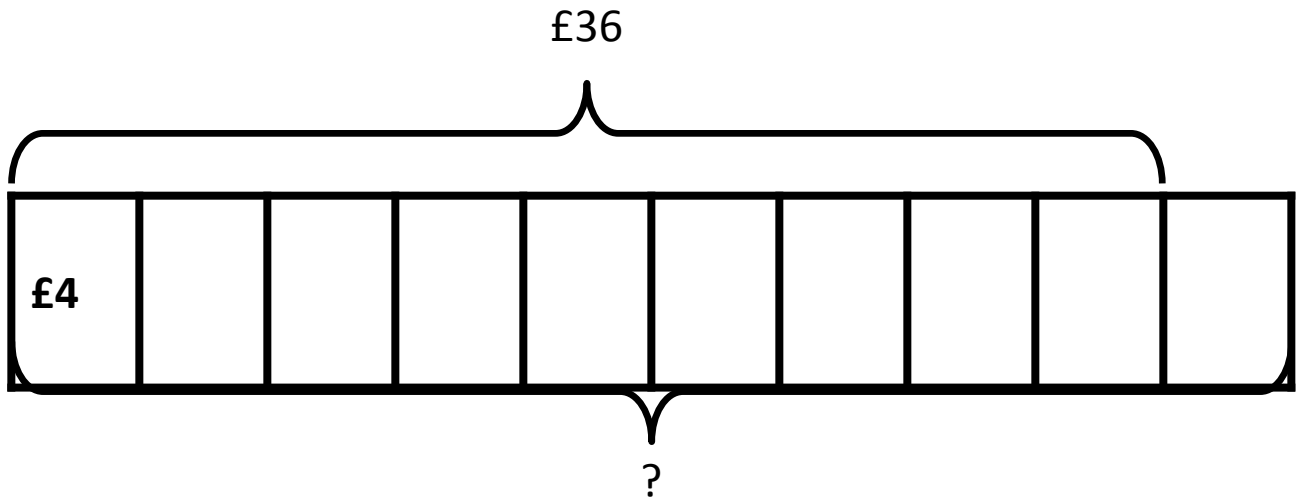
1) How many sections would you shade to show 60%?

2) How many sections would you shade to show $\frac{1}{4}$ of the remainder of the bar?

3)What fraction is left?

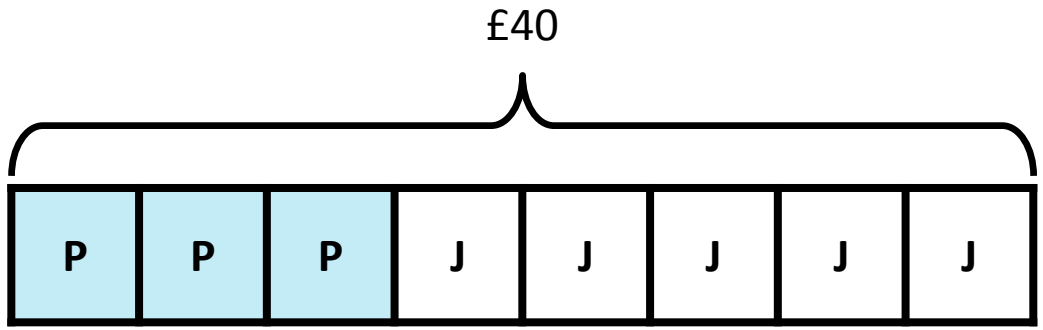
Bar Model 1

- In a 10% off sale, a jacket cost £36.
- What was the full price before the sale?



Use for ratio

- Peter and Jane share £40 in the ratio 3 : 5
- How much does Peter get?



Bar Model 2

- Last year, Jora spent
- 30% of his salary on rent
- $\frac{2}{5}$ of his salary on entertainment
- $\frac{1}{4}$ of his salary on living expenses.
- He saved the rest of his salary.
- Jora spent £3600 on living expenses.
- Work out how much money he saved.

From paper *5MB2H/01, Nov 2010, Q9*

- Jora spent 30% of his salary on rent.
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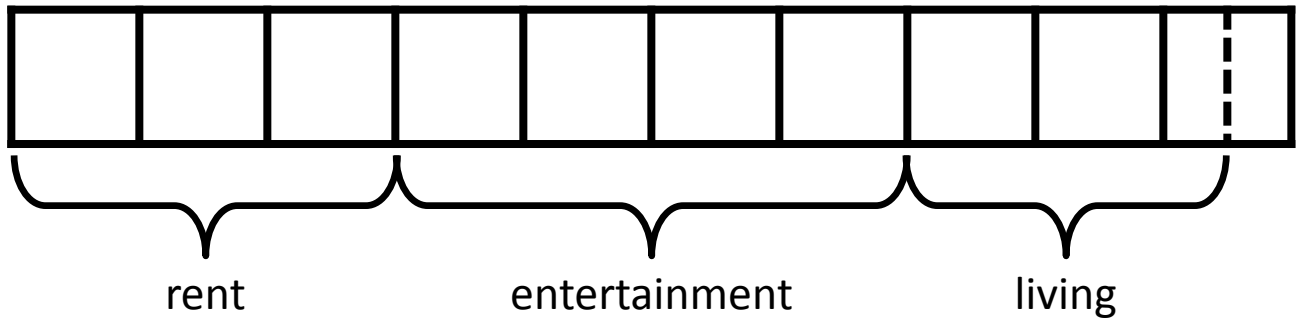

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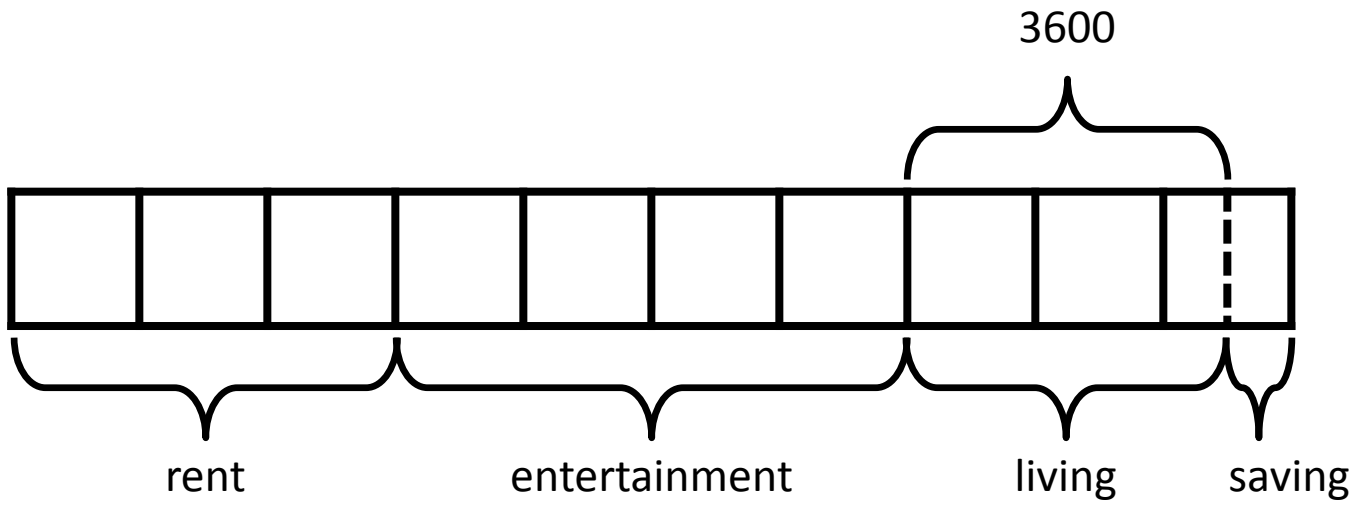


£3600 ÷ 5 = £720

Strategy 3

Make a table to help identify patterns or clues

Look for patterns

Spotting a pattern can help with a wide range of maths problems, including

- Sequences
- Angles in polygons
- Index laws
- Writing expressions and formulae

Looking for patterns

How many hours will a car traveling at 65 miles per hour take to catch up with a car traveling at 55 miles per hour if the slower car starts one hour before the faster car?

Solution

Hour	1	2	3	4	5	6	7
Slower Car	55	110	165	220	275	330	385
Faster Car	0	65	130	195	260	325	390

Look for patterns – example 2

11 × 11	121
111 × 111	12 321
1111 × 1111	1 234 321
11 111 × 11 111	123 454 321

- What is 111 111 x 111 111?

111 111 × 111 111 **12 345 654 321**

Example 1 (Foundation)

One hundred students studying music at school are asked to choose their preference from rap, jazz and classical.

Of the 29 who choose rap, 13 are girls. Of the 21 who choose jazz, 10 are girls.

There are 54 boys altogether.

What percentage of the boys chose rap?

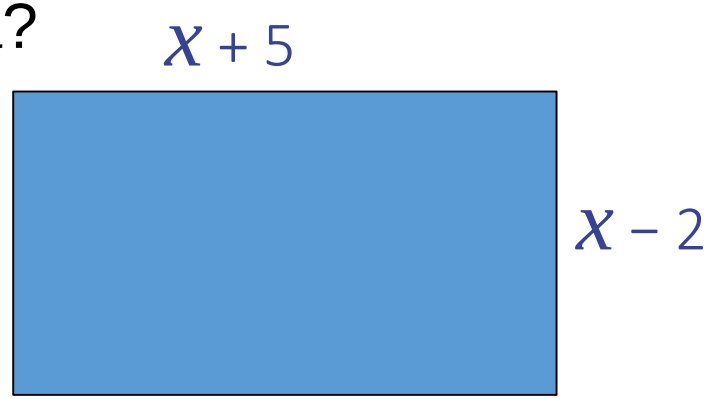
Strategy 4

Form and solve an equation for unknown numbers.

$$
\begin{aligned}
9x + 3 &= 21 \\
-3 \quad & \quad -3 \\
\frac{9x}{9} &= \frac{18}{9} \\
x &= 2
\end{aligned}
$$

Example 1

The perimeter of this rectangle is 66 cm.
What is its area?



$$x + 5 + x - 2 + x + 5 + x - 2 = 66$$

$$4x + 6 = 66$$

$$4x = 60$$

$$x = 15$$

So the dimensions are 20 cm by 13 cm.

The area is $20 \times 13 = 260\ \text{cm}^2$

Using x for the unknown 1

Alice, Beth and Carl buy sweets.

Beth buys twice as many sweets as Alice.

Carl buys 10 more sweets than Beth.

They buy a total of 30 sweets.

How many sweets does Alice buy?

Let the number of sweets Alice buys be x

Beth buys $2x$ sweets

Carl buys $2x + 10$ sweets

$x + 2x + 2x + 10 = 30$

$5x + 10 = 30$

$5x = 20$

$x = 4$

I am 30 years older than my daughter.

My son is 4 years younger than my daughter.

The sum of our ages is 80 years.

How old are we?

Using x for the unknown 2

Let my age be x

My daughter's age is: $x - 30$

My son's age is: $x - 34$

The sum of our ages is 80

$$x + x - 30 + x - 34 = 80$$

$$x + x - 30 + x - 34 = 80$$

$$3x - 64 = 80$$

$$3x = 144$$

$$x = 48$$

My age is 48, my daughter is 18 and my son is 14

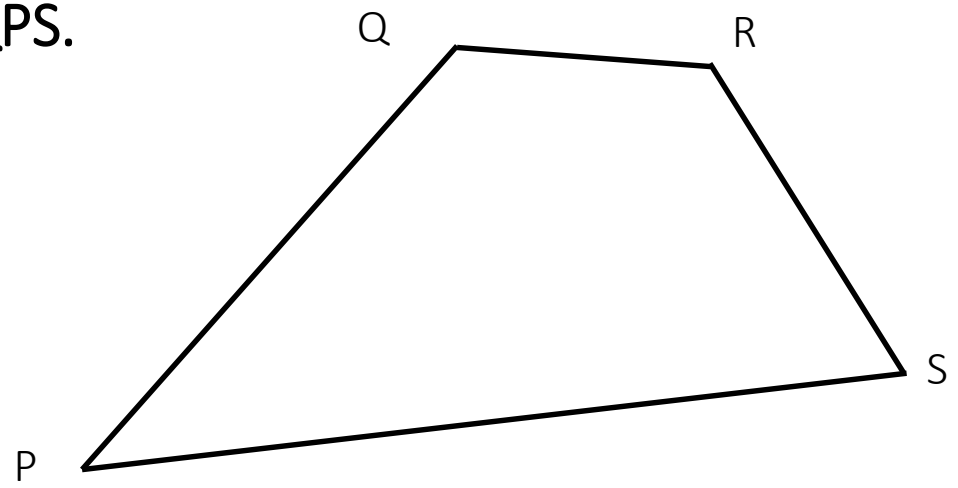
Using x for the unknown 3

In this quadrilateral, angles PQR and QRS are equal.

Angle QRS is 1.5 times the size of angle PSR.

Angle PSR is twice the size of angle QPS.

Find the size of angle QPS.



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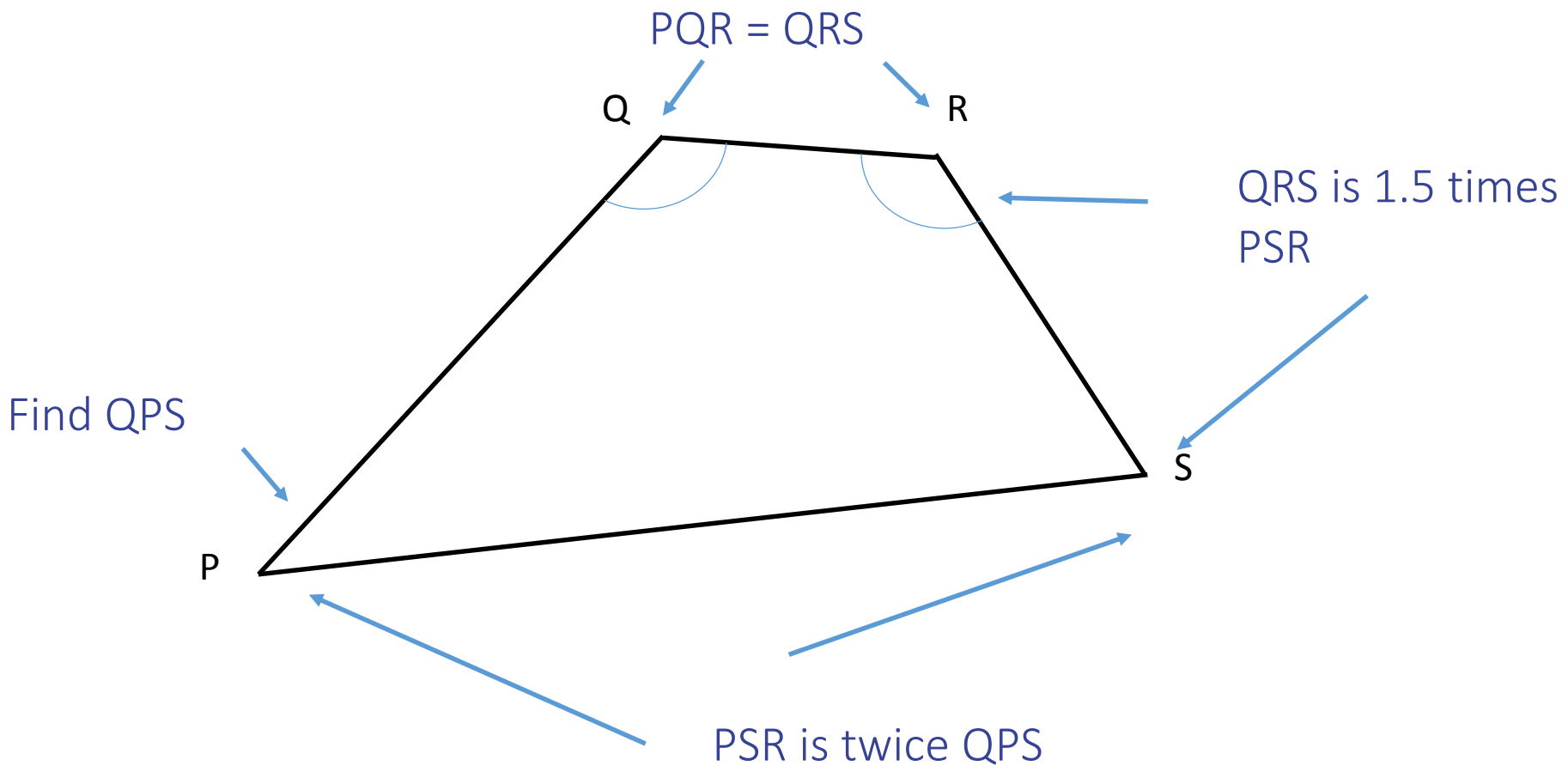

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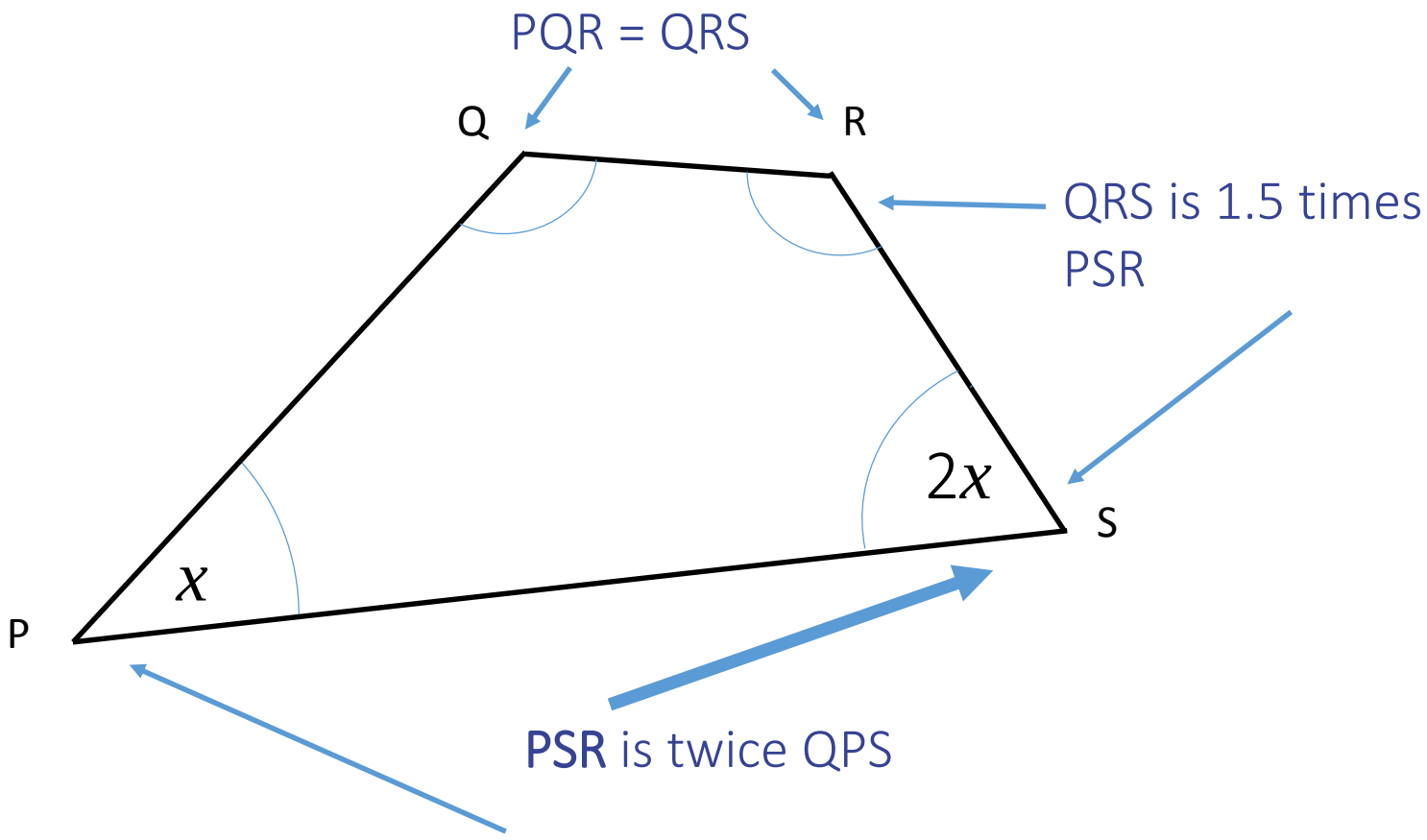

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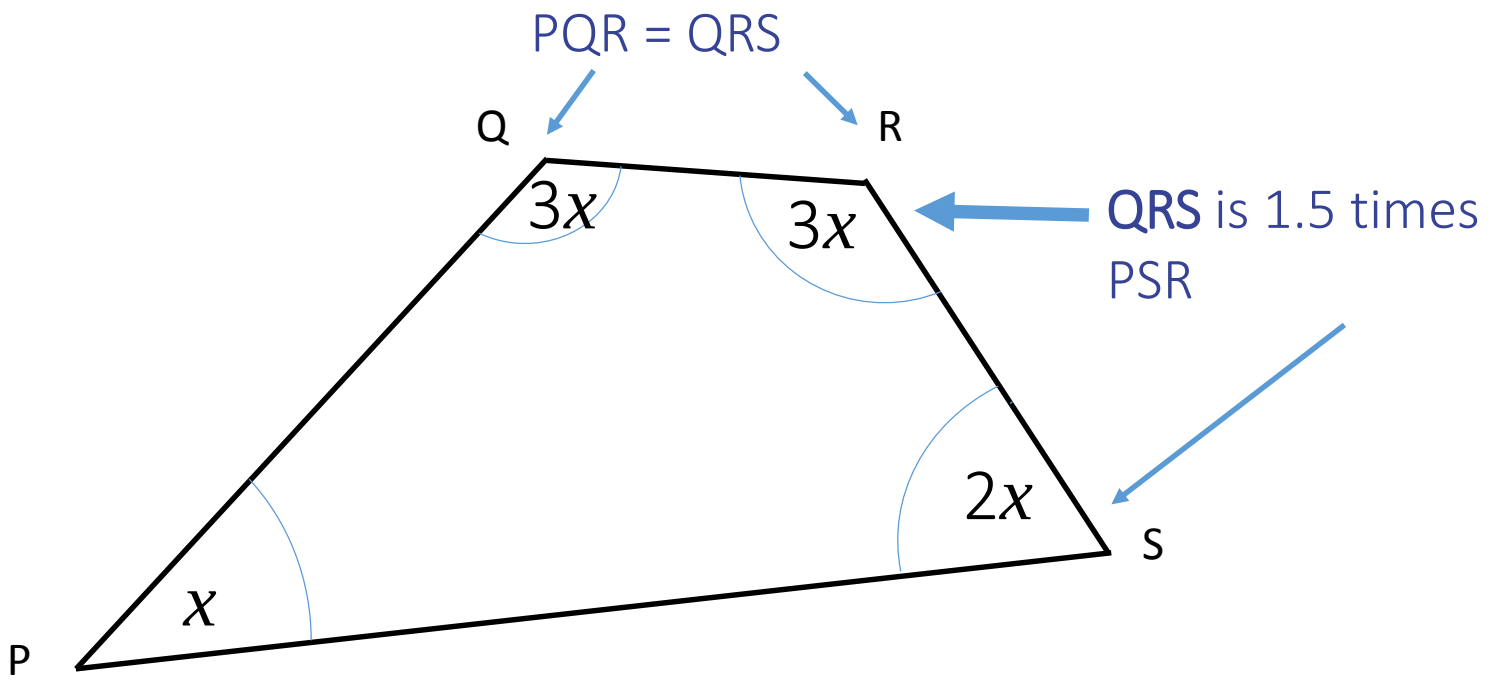

$3x + 3x + 2x + x = 360$

$9x = 360$

$x = 40$

Problem solving strategies recap

- Draw a picture
- Bar models
- x for the unknown
- Look for patterns
- Make a table or a list
- Arrow diagrams (we did not look at this)
- Flow diagrams (we did not look at this)

Using graphic organisers

- **Graphical organisers can be used to help you organise the information and solve the problem.**

- **There is an example of one on the next slide and a copy is also included within your booklet..**

Using graphic organisers

Step 1 What information are you given? *Students need to choose what information they require from the question. This may include words, algebra and diagrams (or words translated into diagrams).*		**Step 2** What do you need to find out? *Students need to decide what it is they are looking for. They may wish to record intermediate steps towards the final answer.*
Reflection & Review *Students need to review their solution. Have they answered the question fully? Does the answer make sense? Is there a better way to do it?*	The question *Write the question here.*	**Step 3** What maths do you need to use? *Students need to decide what mathematical techniques they require to solve the problem.*
	Step 4 Working and solution *Students use this space to record their working and final solution.*	

Using graphic organisers

What information are you given?		What do you need to find out?
Reflection & Review	The question	What maths do you need to use?
	Working and solution	

Problem Solving Strategies Workshop

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....is knowing what to do when you don’t know what to do

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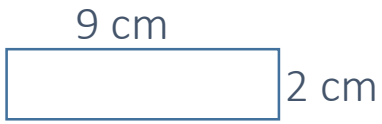

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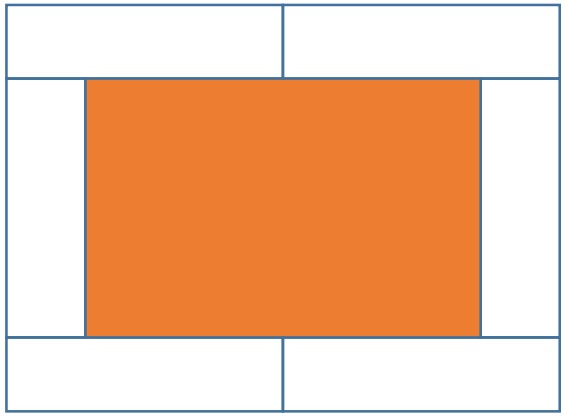

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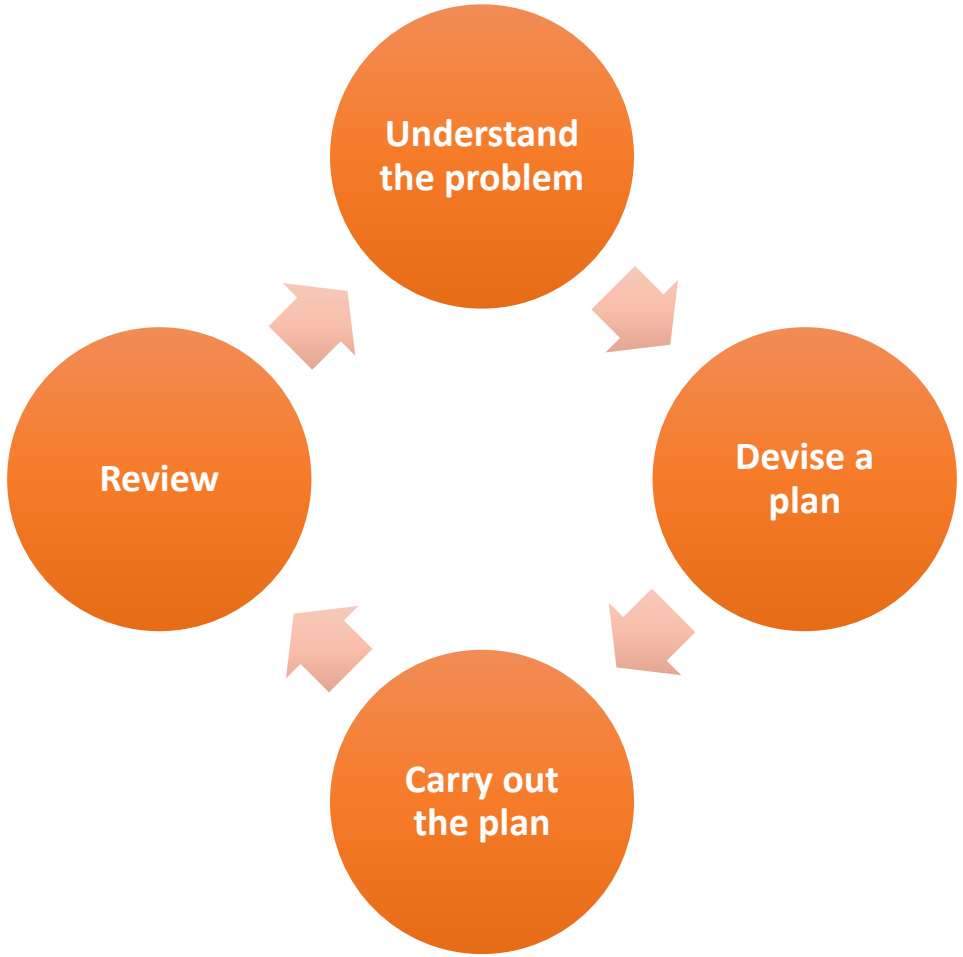
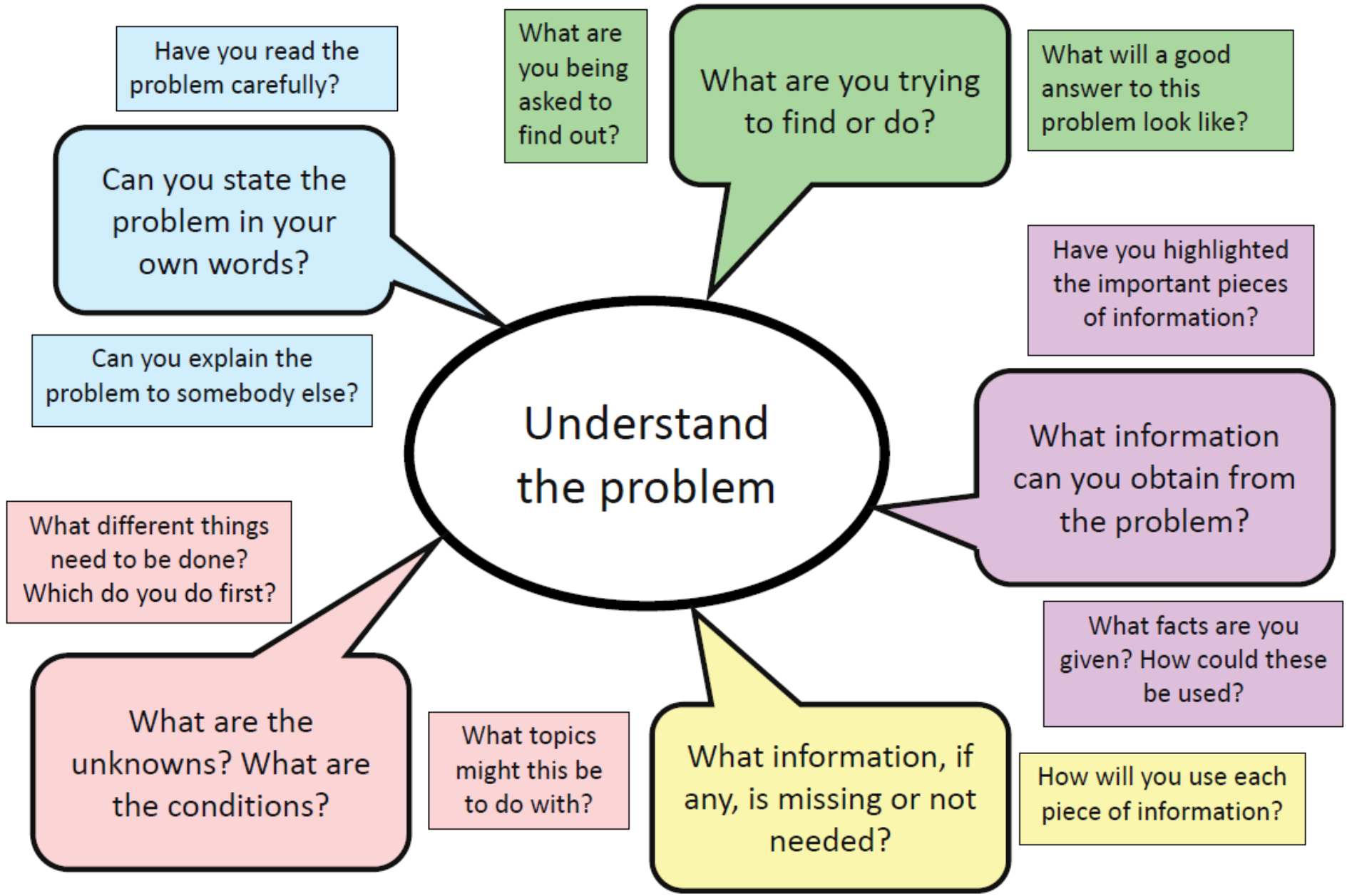
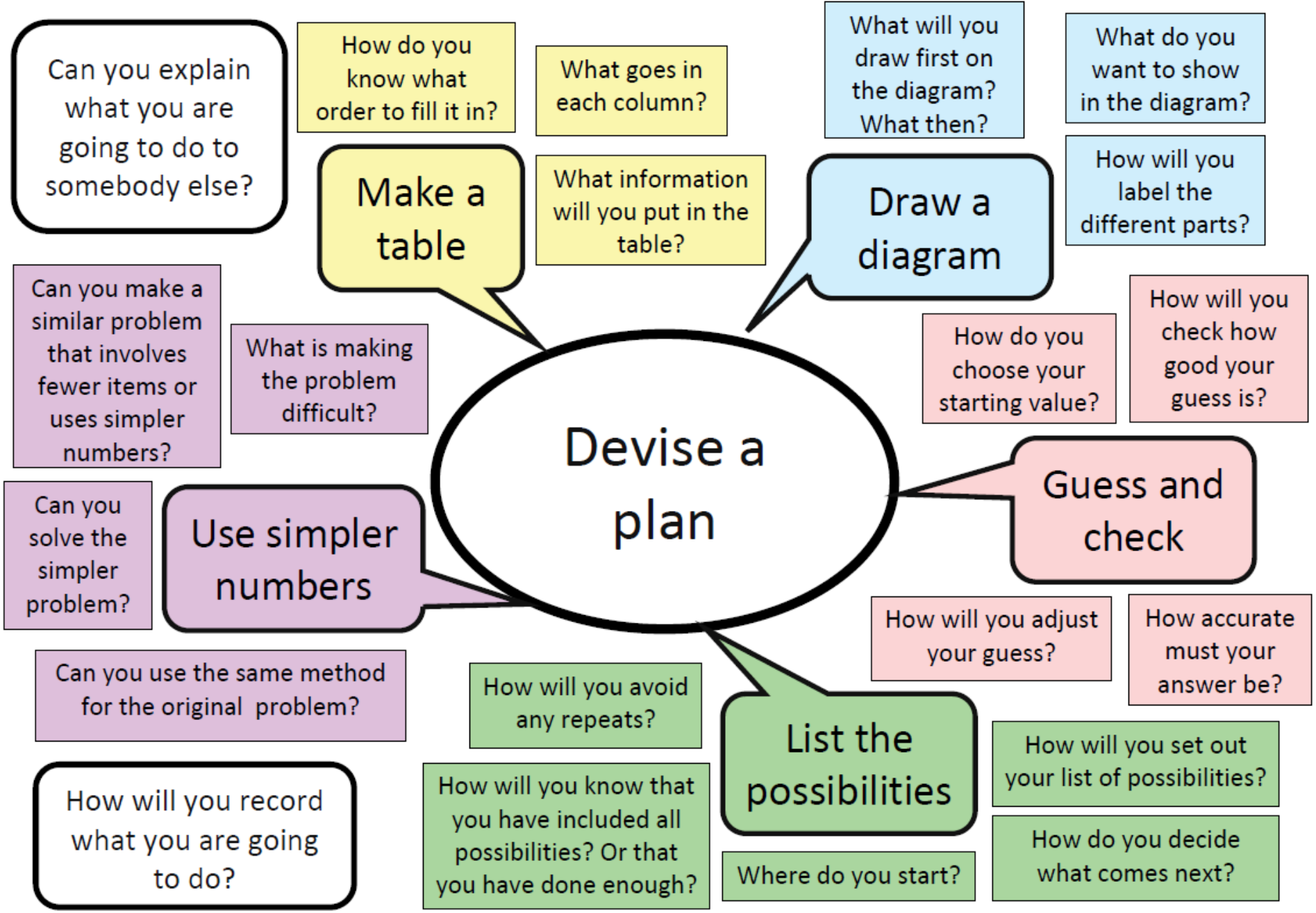
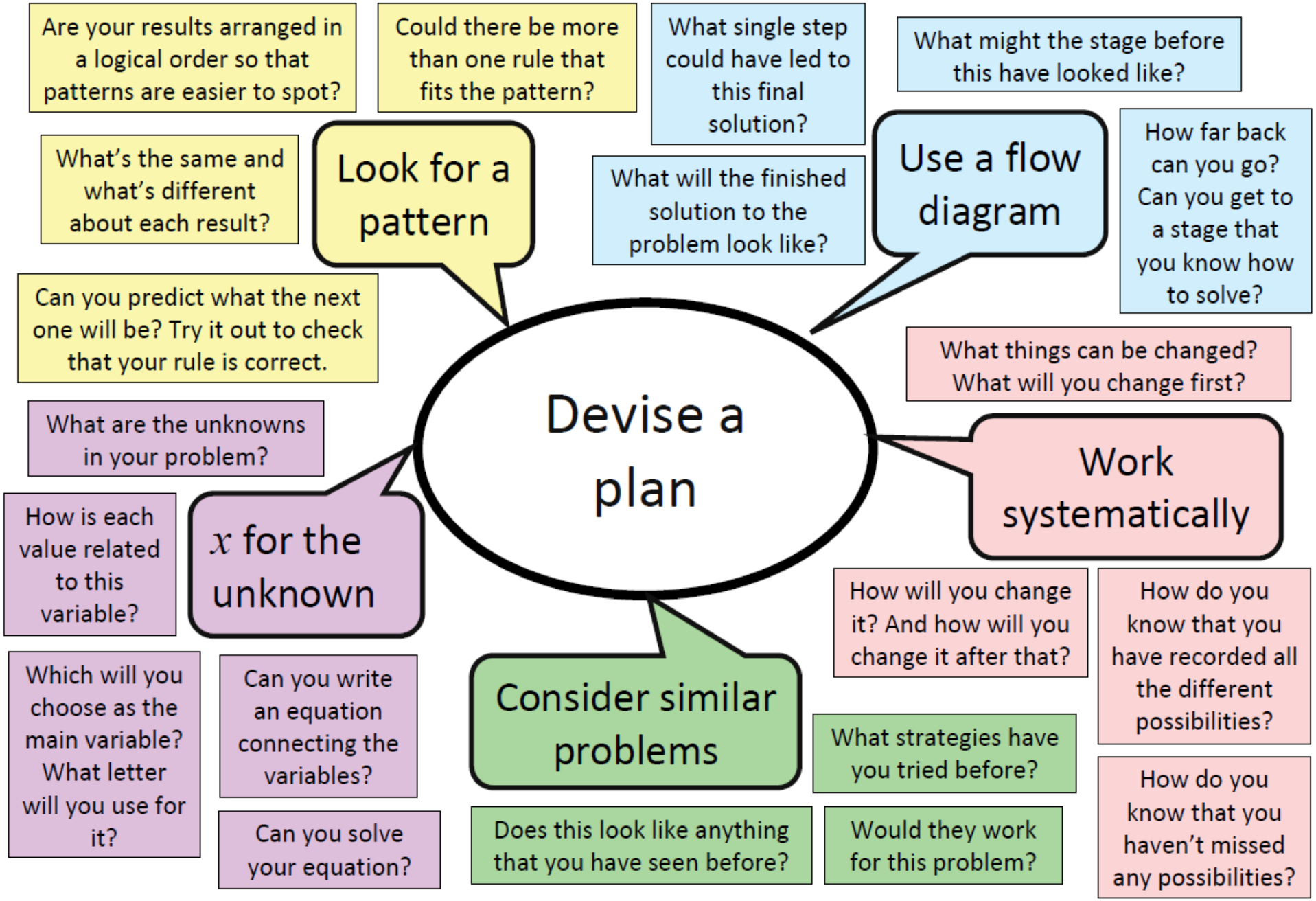
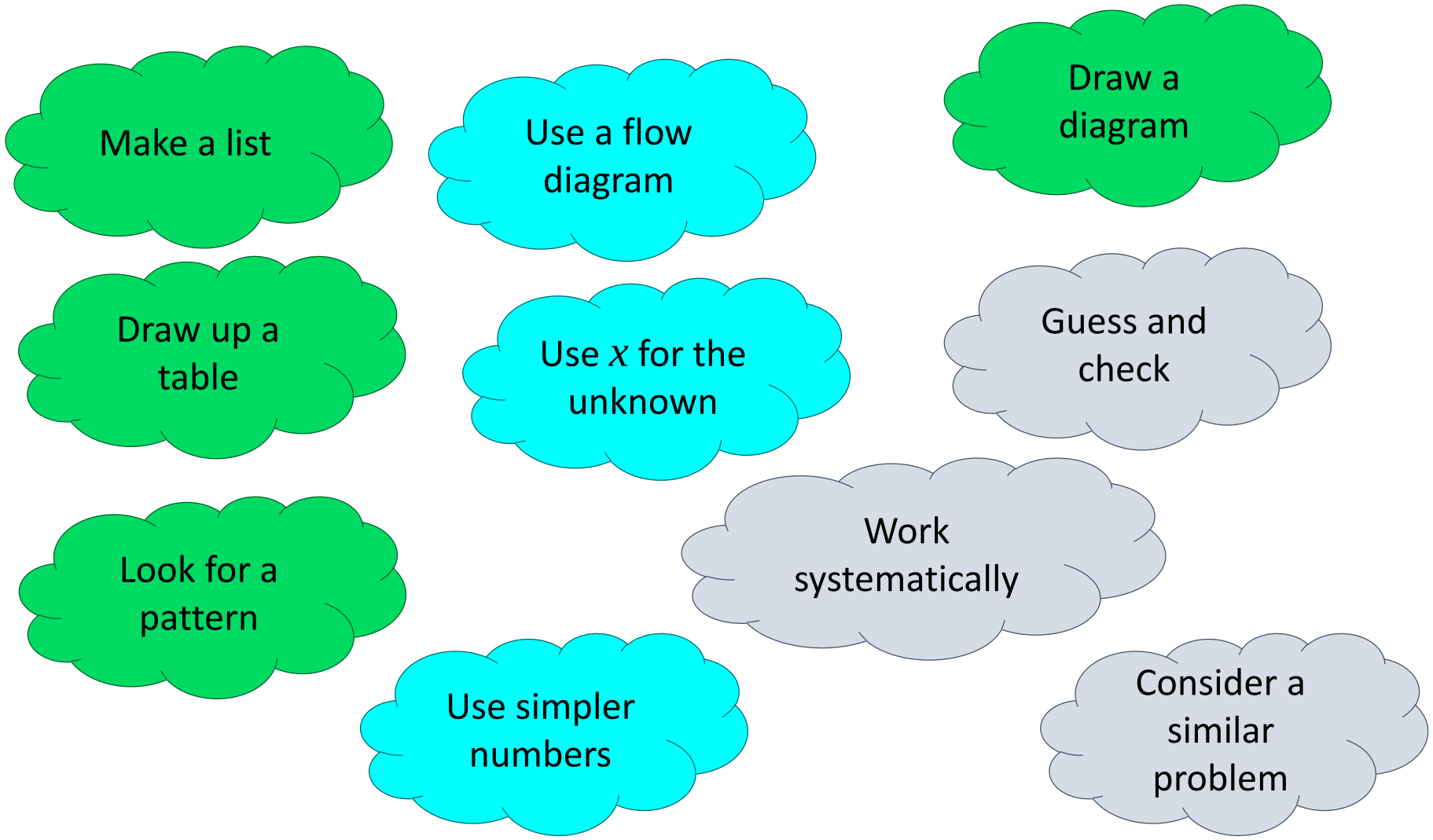
When you have to solve a problem, following these phases will help.

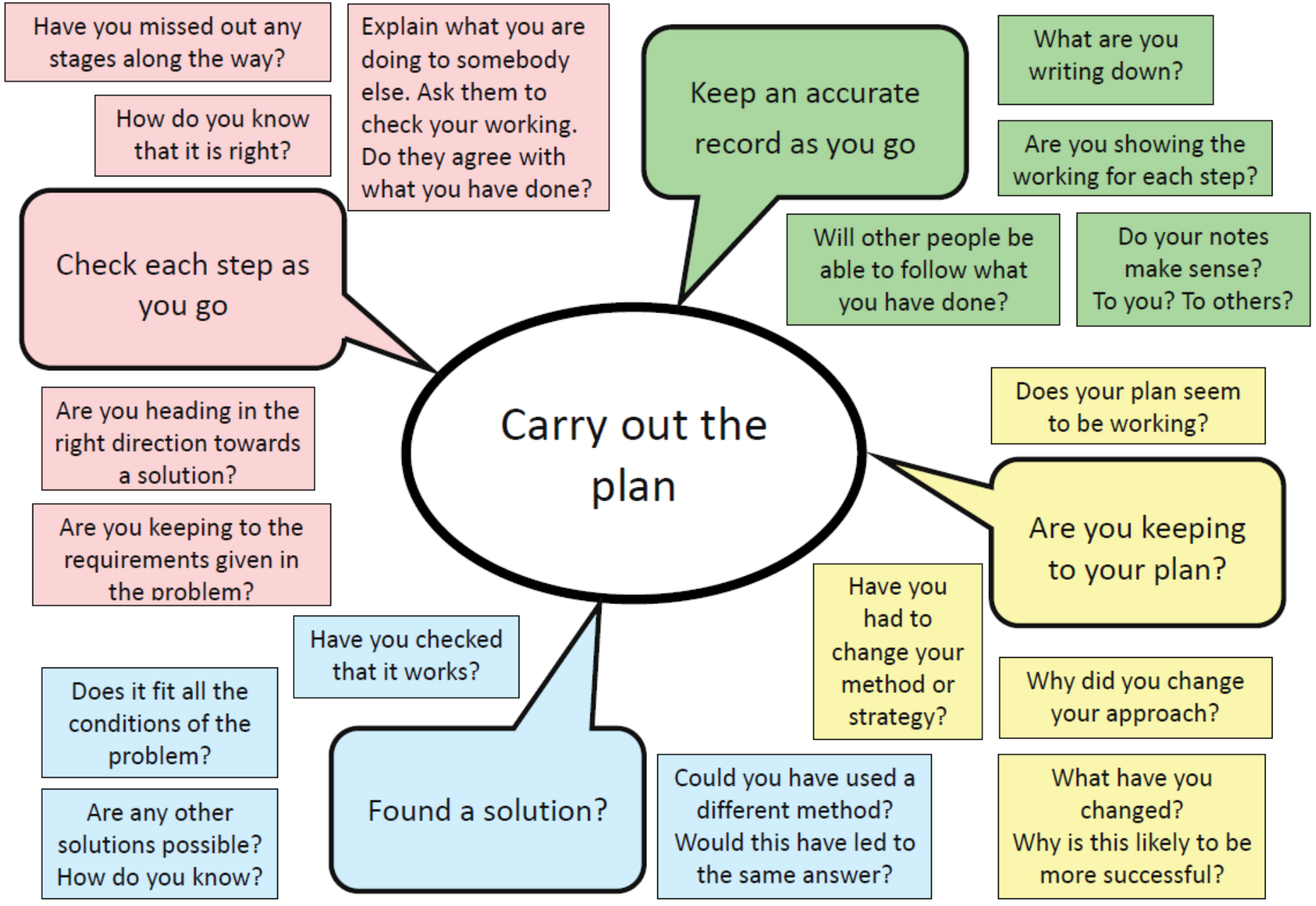


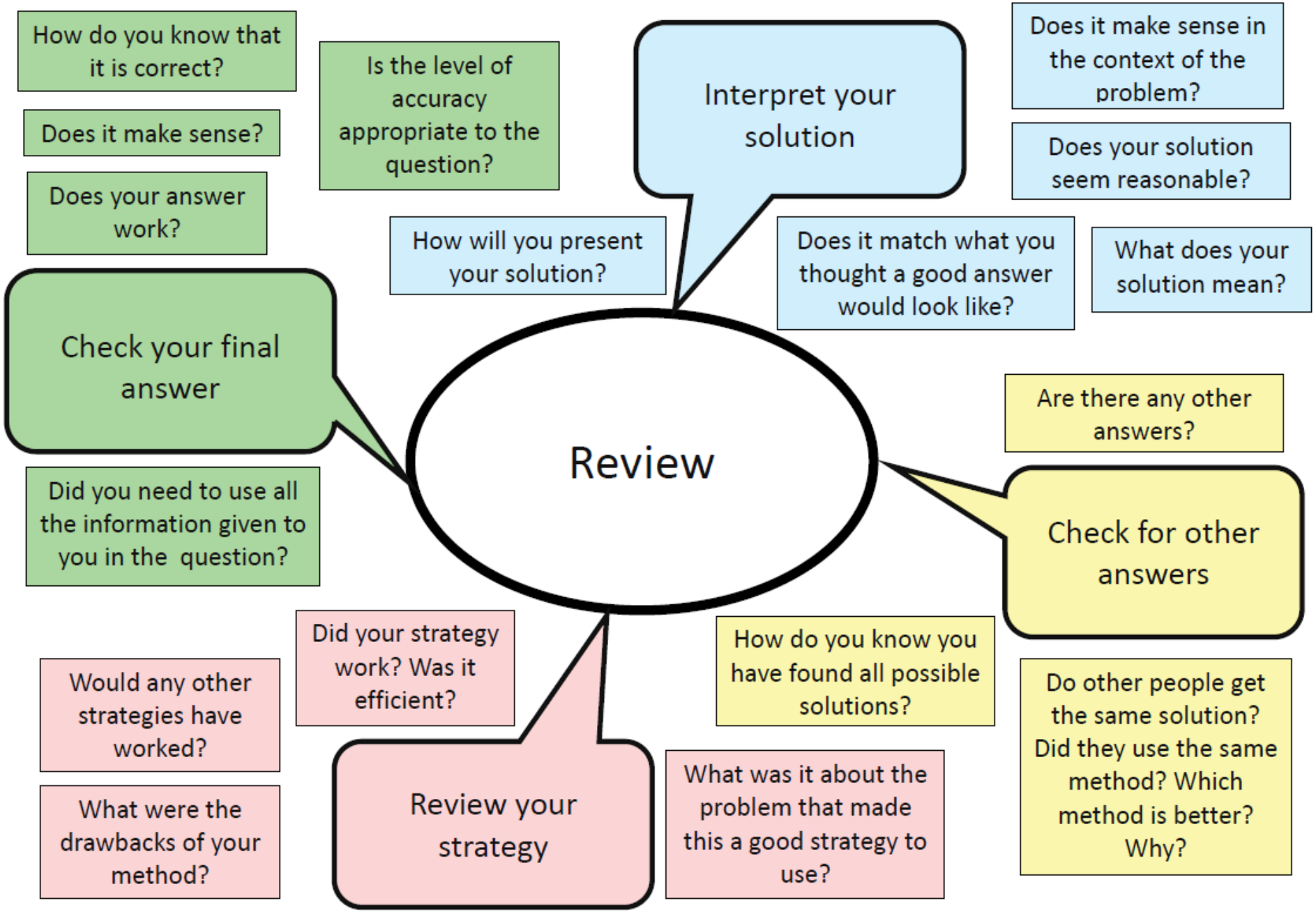

Understand the problem
Have you read the problem carefully?
Can you state the problem in your own words?
Can you explain the problem to somebody else?
What are you being asked to find out?
What are you trying to find or do?
What will a good answer to this problem look like?
Have you highlighted the important pieces of information?
What information can you obtain from the problem?
What facts are you given? How could these be used?
How will you use each piece of information?
What information, if any, is missing or not needed?
What topics might this be to do with?
What are the unknowns? What are the conditions?
What different things need to be done? Which do you do first?


Devise a plan
Make a table
How do you know what order to fill it in?
What goes in each column?
What information will you put in the table?
Draw a diagram
What will you draw first on the diagram? What then?
What do you want to show in the diagram?
How will you label the different parts?
Guess and check
How do you choose your starting value?
How will you check how good your guess is?
How will you adjust your guess?
How accurate must your answer be?
List the possibilities
How will you avoid any repeats?
How will you know that you have included all possibilities? Or that you have done enough?
Where do you start?
How will you set out your list of possibilities?
How do you decide what comes next?
Use simpler numbers
Can you make a similar problem that involves fewer items or uses simpler numbers?
What is making the problem difficult?
Can you solve the simpler problem?
Can you use the same method for the original problem?
Can you explain what you are going to do to somebody else?
How will you record what you are going to do?


Are your results arranged in a logical order so that patterns are easier to spot?
Could there be more than one rule that fits the pattern?
What single step could have led to this final solution?
What might the stage before this have looked like?
What's the same and what's different about each result?
Look for a pattern
What will the finished solution to the problem look like?
Use a flow diagram
How far back can you go? Can you get to a stage that you know how to solve?
Can you predict what the next one will be? Try it out to check that your rule is correct.
What things can be changed? What will you change first?
Devise a plan
What are the unknowns in your problem?
How is each value related to this variable?
x for the unknown
Work systematically
How will you change it? And how will you change it after that?
How do you know that you have recorded all the different possibilities?
Which will you choose as the main variable? What letter will you use for it?
Can you write an equation connecting the variables?
Consider similar problems
What strategies have you tried before?
How do you know that you haven't missed any possibilities?
Can you solve your equation?
Does this look like anything that you have seen before?
Would they work for this problem?


Make a list
Use a flow diagram
Draw a diagram
Draw up a table
Use x for the unknown
Guess and check
Look for a pattern
Work systematically
Use simpler numbers
Consider a similar problem


Carry out the plan
Check each step as you go
Have you missed out any stages along the way?
How do you know that it is right?
Explain what you are doing to somebody else. Ask them to check your working. Do they agree with what you have done?
Keep an accurate record as you go
What are you writing down?
Are you showing the working for each step?
Will other people be able to follow what you have done?
Do your notes make sense? To you? To others?
Are you keeping to your plan?
Does your plan seem to be working?
Are you heading in the right direction towards a solution?
Are you keeping to the requirements given in the problem?
Have you had to change your method or strategy?
Why did you change your approach?
Found a solution?
Have you checked that it works?
Does it fit all the conditions of the problem?
Are any other solutions possible? How do you know?
Could you have used a different method? Would this have led to the same answer?
What have you changed? Why is this likely to be more successful?


Review
Interpret your solution
Does it make sense in the context of the problem?
Does your solution seem reasonable?
What does your solution mean?
Does it match what you thought a good answer would look like?
How will you present your solution?
Check your final answer
How do you know that it is correct?
Is the level of accuracy appropriate to the question?
Does it make sense?
Does your answer work?
Did you need to use all the information given to you in the question?
Check for other answers
Are there any other answers?
How do you know you have found all possible solutions?
Do other people get the same solution? Did they use the same method? Which method is better? Why?
Review your strategy
Did your strategy work? Was it efficient?
What was it about the problem that made this a good strategy to use?
Would any other strategies have worked?
What were the drawbacks of your method?

Drawing a Diagram 1

Draw a diagram to help solve this problem.

A circus clown climbs a 16 m slippery pole.

When the music plays, he climbs up 4 m.

When the music stops, he slides down 2 m.

How many times must the music play for the clown to reach the top of the slippery pole?

Drawing a diagram 2

In a café, tea costs £1.40 and hot chocolate costs £1.80.
A group of friends bought some drinks and paid £10.
How many of each kind of drink did they buy?

Bar Model 1

- In a 10% off sale, a jacket cost £36.
- What was the full price before the sale?

Bar Model 2

- Last year, Jora spent
- 30% of his salary on rent
- $\frac{2}{5}$ of his salary on entertainment
- $\frac{1}{4}$ of his salary on living expenses.
- He saved the rest of his salary.
- Jora spent £3600 on living expenses.
- Work out how much money he saved.

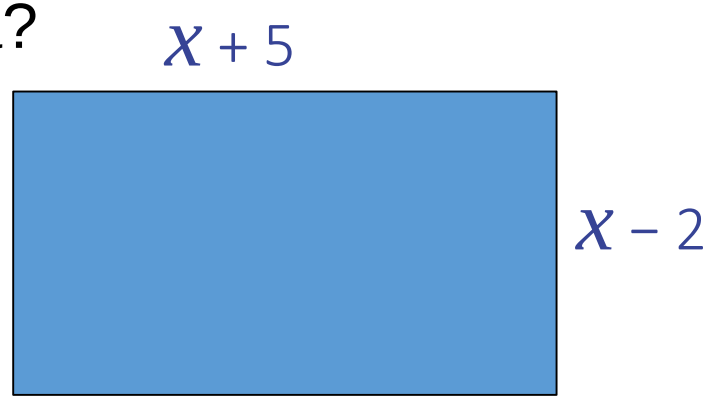
From paper *5MB2H/01, Nov 2010, Q9*

Looking for patterns

How many hours will a car traveling at 65 miles per hour take to catch up with a car traveling at 55 miles per hour if the slower car starts one hour before the faster car?

Example 1

The perimeter of this rectangle is 66 cm.
What is its area?



I am 30 years older than my daughter.

My son is 4 years younger than my daughter.

The sum of our ages is 80 years.

How old are we?

Using x for the unknown 2

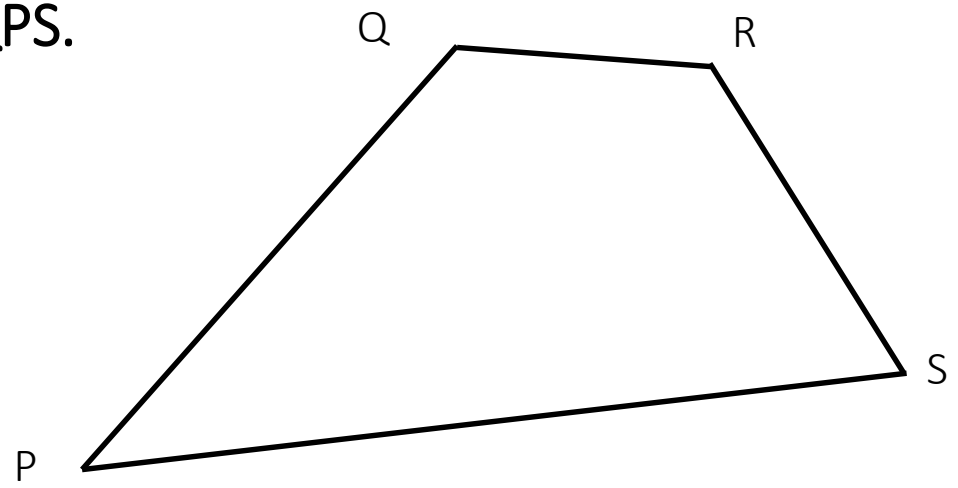
Using x for the unknown 3

In this quadrilateral, angles PQR and QRS are equal.

Angle QRS is 1.5 times the size of angle PSR.

Angle PSR is twice the size of angle QPS.

Find the size of angle QPS.



Using graphic organisers

What information are you given?		What do you need to find out?
Reflection & Review	The question	What maths do you need to use?
	Working and solution	